TEACHING & LEARNING RESEARCH INITIATIVE
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Teachers Learning Mathematics: Professional development research. A Summary

Introduction

The issue of mathematical knowledge of teachers has been documented in New Zealand for 80 years. For example, the 2004 New Zealand Ministry of Education Teacher Census showed that 25 percent of secondary mathematics teachers had no university mathematics qualification—a rise from 21 percent in 1977.

This one-year study aimed to investigate the development of teachers' own mathematical knowledge for teaching. Seven secondary teachers from the Auckland region each developed some aspect of their mathematical knowledge. Two external researchers supported the teachers and facilitated reporting.

Aims and objectives

The research questions were as follows:

1. What developments will a teacher be able to make in their own mathematical knowledge while teaching?
2. What do these teachers think about its worthwhileness and practicality as ongoing professional development?
3. What do these teachers have to say about the importance of the mathematical knowledge of a teacher?

A subaim of the project was to further induct the teacher-researchers into research practice.

The project sought to understand how teachers can engage in the learning of mathematics to enhance their teaching as part of their professional lives. It gave a group of teachers the opportunity to undertake such learning in a supported fashion, and to reflect on and investigate for themselves its effect.

Research design

This study drew on three areas: theories about mathematical knowledge needed for teaching; teacher research as a means of professional development; and the nature of professional development generally. The themes of teacher control and professional communities of teachers were both important in the design of the project.

The methodology for the individual investigations was based on action research. Two cycles were attempted by each teacher-researcher, all completing at least one.

Each teacher was asked to focus on a mathematical topic that either: they had found difficult to understand themselves; or they understood but felt they had not taught effectively; or they felt they had taught effectively, but the students found difficult.

With a university researcher, each teacher planned a mathematical development scheme for this topic, including data collection and analysis concerning the influence on themselves or their teaching.

The collective part of the research was naturalistic in its methodology. The teacher-researchers researched themselves as a group, using their own recorded experiences.

The project

The group first met in December 2006 to establish each teacher's area of interest. Three meetings were held in February, May and June of 2007. By July each teacher-researcher had completed Cycle 1, and all teachers had been able to introduce aspects of their learning into their classes. Cycle 2 entered the planning stage and a joint presentation for the New Zealand Association of Mathematics Teachers (NZAMT) conference was developed. The teacher-researchers kept records of their own progress, and university researcher notes from the meetings were returned to the teacher-researchers for elaboration. During the first half-year we had two video-linked meetings with the Oxford University team.

In the second half of 2007 teachers worked on a second phase of the same topic. Support continued in a similar way. The teacher-researchers shared their findings with their school mathematics departments.

Two preparatory meetings were held for the joint NZAMT presentation which took place in late September to an audience of 25 teachers. We received positive and interested verbal feedback after the presentation, two teachers asking directly to be able to be involved in further similar work.

In November all the teacher-researchers' reports were finalised.

Project findings

First, the individual teacher-researchers' experiences are briefly described, then the role of the group as a professional learning community, and, finally, the role that increased understanding of mathematical knowledge plays in effective teaching.

Individual experiences

Anne Blundell

Anne began studying proof. During university she struggled with proofs and reported that "it was with trepidation that I set out to correct a personal weakness". She began discussing proofs with colleagues and tentatively introducing them into her teaching. Later in the year she introduced more proofs.

She made it clear to her students that she had become a learner, and chronicled how she had changed "from proof-o-phobe to proof-o-phile". As the year progressed "I would not settle for anything less than a full explanation".

Anne found her work on proofs linked with other professional development; for example, with training on Excel. She considers that she still has a long way to go in developing proofs in her teaching.

Linda Crisford

Linda focused on a better understanding of trigonometry, an area in which she was very weak. She began by asking other teachers about their enthusiasm for mathematics, and discussed ways teachers could continue to feel enthused about their subject.

Linda's mode of engaging her colleagues may well be a key generalisable finding in this project. She summarises:

> Because the teachers in my department are so busy I found that they were happy to assist me so long as I was fast; that is 'Can you just show me how to do this in four minutes. If I don't get it in four, forget it.' This meant I got seven to 10 minutes of one-to-one explanation. I asked to be part of their class silently for me to learn trig or anything they were teaching.

Margaret de Boer

Margaret's focus on logarithms was driven both by her students' assessment needs and her realisation that there are "a lot of applications of logs, and I didn't understand them well enough". She originally refers to the log laws as, "these three or four little rules" and was surprised that students could not learn them.

She began by exploring their history. She introduced historical detail and the development of logarithms into her lessons. She reports that while the students "weren't really following the process in the same way I was, they did get something from it".

Margaret became aware of the depth of conceptual connections underlying an understanding of logarithms and of the way an understanding of history can inform one's pedagogy.

Anna Dumnov

Anna wanted to introduce more history into her teaching. She focused on algebra and *e*, the base of natural logarithms. She read history, and introduced her learning into her class.

In the second cycle Anna worked alongside a student who was developing her understanding of *e*. The student made *e* the subject of her presentation to a group of seniors at the school.

Anna "wanted students to see that not only was I learning new historical information, but that this information leads me into a new mathematical context".

Jason Florence

Jason initially attempted to study university-level algebra, but, after being initially enthusiastic, he found it a lonely process. Also, he could not find a way to introduce the ideas to his students in a meaningful way.

The second phase of his learning was prompted by being part of a statistics education group, and he began to work on probability, developing an extensive unit of work on heuristics in decision making. He subsequently presented this work at the Auckland Mathematics Association (AMA) statistics day.

Peter Radonich

Peter was interested in mathematical modelling and connections between drawing and working mathematically. His explorations were inspired by Peter Hunter's work on the heart, Len Lye's creations and by building models using LOGO. He then focused on developing an approach to algebra in which students developed a "family" of creatures whose limbs increased according to a pattern. Students responded enthusiastically.

He is now interested in the geometry of the HIV virus, and the role 3D geometry software may play in learning mathematics.

Yoko Raike

Yoko's work was a series of iterative cycles, focusing on why differential equations are useful. After a meeting with a mathematical physiologist, she looked at differential equations that model the behaviour of HIV. She reported that rate became a more central concept in her teaching at all levels. She did four presentations based on her work.

Yoko reflects:

> I wondered why I did not develop this understanding of the usefulness of differential equations when I was a learner. My conclusion is that I focused only on how to solve equations. The rewards of learning were success in examinations.
>
> I believe that it is worth explaining the usefulness of differential equations in class and a teacher having 'fun' with the topic must have an impact on her teaching.

In 2008 she is going to continue her studies of mathematics and disease through courses at university.

Summary of individual experiences

A number of themes emerge. The value of:

- communication, openness, connecting people and ideas and finding time for personal mathematical development
- working in a domain identified by the teacher as valuable and about which they may be anxious
- students seeing their teachers as learners
- stimulation through reading, or talking to experts
- teachers' thirst for learning more mathematics.

We have realised again how important trust is in the learning situation, and how sensitively people who are exposing their vulnerabilities need to be handled.

We have questions about how to measure the change that the teachers report, and whether there is a negative side to teacher enthusiasm.

Role of the group as a professional learning community

Teacher-researchers attribute much of their mathematical learning to collegiality. Not only was there a transfer of knowledge between teachers, but learning together also helped teachers progress. Although it had always been possible for the teachers to learn mathematics, it was only when they joined together that they did so.

Several report insecurity about approaching mathematical topics. In all cases the group was a significant means of moving on.

Several report their desire to continue working in such an environment. There were no negative responses.

This study further confirms the need for a professional community in professional development. Teacher-researchers used their school groups less than expected, and more research needs to be done to understand why this is so. Also, there was no noticeable effect on participation in the wider professional community. Perhaps this community is too big and meets too infrequently to provide the support needed.

The model of professional development was highly successful. It is difficult to know how far this conclusion can be generalised.

We hypothesise some key elements, but further research is needed:

- trust between teachers
- the link with university-level mathematicians
- teacher control over professional development
- external support.

Role of teachers' mathematical understanding in effective teaching

Teacher-researchers give examples of their learning directly affecting their teaching practices. We have identified three characteristics. They are the increased variety of mathematical learning opportunities offered to students, the effect of students seeing their teachers as learners and teachers thinking deeply about their classroom practice.

The study was not designed to assess impacts on student learning, and we believe that this needs a large-scale long-term study. The potential for improvement in

learning is most likely to emerge from changes to the way teachers think about particular aspects of their teaching. We hypothesise that maintaining a professional community over a long period will help to ensure that the change in perspective continues to develop.

Providing particular new learning opportunities is likely to be the most unstable of the characteristics. However, evidence from the literature is clear that experiences that are richer *mathematically* are more likely to produce better learning.

The future

The teachers wish to continue to study mathematics for themselves, making use of the methods they developed during the project—reading, exploring areas they find problematic, continuing their exploration of areas of interest, working with colleagues, studying Master's papers and attending short courses.

Learning communities were set up within the project group and in some of the teachers' departments. However, school-based communities change in response to local priorities. Whether the groups will be sustained without regular meetings and outside input remains to be seen.

Limitations

This was a small-scale study with good secondary teachers in supportive and mid- to high-decile schools who were familiar with the university researchers. This raises questions about whether the results would be the same with a group different in any of these characteristics.

We believe that a larger group of similar teachers would have similar results. A teacher who was struggling in class, or who did not trust the external staff, would be unlikely to be as open with their weaknesses and as willing to address them.

Our collective experience of low-decile schools makes us conclude that it is unlikely the results would be the same, as they do not have the personal conditions necessary to undertake this type of activity.

Capacity building

A number of levels of capacity building resulted from this project:

- The teacher-researchers will continue developing their mathematics content knowledge.
- The teacher-researchers are better potential leaders.
- The teacher-researchers are better able to undertake future research.
- The university researchers are better able to continue research in this area.
- This practical model of professional development is now known to be effective.

Research outputs and disseminations

There were three components of dissemination:

(a) Teachers presented their own studies and findings to teachers in their departments.

(b) A joint presentation at the NZAMT conference.

(c) Scholarly paper (in process).

Recommendations

We have three recommendations:

1. The establishment of a professional community of teachers needs to be a vital component of all professional development interventions.
2. Teachers should be provided with opportunities to gain ongoing mathematical stimulation.
3. Programmes should be established to link teachers in schools with university (or workplace)-based practitioners of their subject matter.

The full reports of all TLRI projects are published on the TLRI website (www.tlri.org.nz).

Lead authors and researchers

Judy Paterson was a high school mathematics teacher in South Africa and New Zealand for 25 years before becoming involved in the education of secondary mathematics teachers in 1997. Initially involved in preservice education in a programme offered jointly through the Department of Mathematics and the School of Education at The University of Auckland, over the last few years Judy has become increasingly interested in inservice professional development—the subject of her doctoral thesis completed in 2007. Judy is interested in how teachers' knowledge of mathematics affects their teaching decisions, and in developing their own mathematics can impact on the energy teachers bring to the classroom.

Bill Barton joined the Mathematics Education Unit in the Department of Mathematics, The University of Auckland in 1993 after 15 years of secondary teaching, including Māori/English bilingual classes. He is currently head of the department. His research interests are in the culture and linguistic issues in mathematics and mathematics education. His book *The Language of Mathematics* was published in 2008. Bill has been elected Vice-President of the International Commission on Mathematical Instruction (ICMI).